



I-D3-1

ERN LOGIC

Foreword

In the chapter "NATURAL MODEL" we have described a particular type of reflection, which we called "Inference", which maps events ordered by causality into symbolic network structures of expressions related "deductively" by Implication, shortly "ERN structures" or "theories". Inverse operation regresses "inductively" expressions to their territory of events or "facts". Deductive theory completed with factual induction will be called a "model". Due to intuiting causality as unshakably "real", deduction appears as "certain" and a theory consistent with deductive rules as "necessary". Induction, on the contrary, retrieving the originating events of symbolic expressions, gets affected by their fuzziness. A model is fuzzy and inductively verifiable/falsifiable.

Mind's faculty to support inference's ERN structures and functions will be called intrinsic or natural "Logic". It is the subconscious Mind's system used instinctively as support of daily behavior. Irreplaceable as pilot of simple activities, it may nevertheless be misleading owing to its inadequacy to handle complex cases due to Mind's limited working memory and incapacity to concentrate simultaneously on numerous issues, as well as to the fuzziness of induction.

Facing shortcomings of their natural faculties, humans usually produced compensating tools: hammer to assist striking and extrinsic "Logical Systems" to assist intrinsic Mind's inference.

I-D3-2

Our ERN logic, as any extrinsic logical system, may be justified exclusively by its capacity to support Mind's intrinsic, ERN logic. Its applications have so far stood this test. It replaces consistently and simply the ill founded noumenal Predicate (pseudo-)Logic.

ERN is an extrinsic Expression/Relation Network structure. Its vertices or nodes are expressions valued by plausibility, a continuous variable ranging from 0 to 1, replacing in Fuzzy System the binary 1/0 (true/false) variable of Exact Systems. Its edges are Relations.

For conciseness' sake we shall call lower level neighbors of a node its "parts" and higher level ones - its "aggregates".

From the point of view of Fuzzy Logical Calculus, nodes are operands and edges - operators. Operators evaluate the plausibility of an aggregate inductively, in function of its parts. Structure, dimensionality of nodes and types of operators are similar to those described in the section "N DIMENSIONAL PROPOSITIONAL CALCULUS" of the chapter "BOOLEAN SUPPORT OF ERN LOGIC", with the continuous fuzzy plausibility replacing the binary logical variable (true/false, or 1/0). Fuzzy operators relating inductively the nodes involve quite complex algorithms (see APPENDIX).

Inference

Rational inquiry reposes essentially on Inference exerted explicitly or implicitly over ERN-like structures.

Inference, a "two-way" procedure encompasses Deduction and Induction scanning ERN respectively top-down and bottom-up.

I-D3-3

Deduction scans the ERN top-down creating a concrete instance of the hypothetical Theory and setting fuzzy operators in relations according to Theory definition. Top nodes having no aggregates (deductive premises), thus nothing to be deduced from, are Axioms, granted as certain, but subject to inductive, eventual or "factual" verification (see below).

Middle nodes, having both, aggregates and parts are Theorems.

Induction scans the ERN bottom-up setting fuzzy Plausibility of nodes in function of their parts (inductive premises) and connecting relation edges (fuzzy Operators), It starts with bottom nodes which, having no parts, thus nothing to be induced from, are set by extralogical events, or "Facts". Inductive scan verifies or falsifies ERN's Theorems and Axioms in the light of facts.

I-D3-4

Diagnose, tutorial

This paragraph presents a tutorial example of a particular ERN program written for a Company which used it to identify malfunctions in space crafts and to suggest remediations.

It's expressed in oversimplified terms of medical diagnose, more familiar to the average reader of the tutorial. The form (indented printouts, etc) is not inherent in ERN, but pertains to the tutorial.

Step 1, Schema

"Schema" designates the general, deductive structure of the theory expressed in types of operands-expressions. In the case of our "diagnose" theory "diagnose" recursively implies "sickness", "syndrome" and "symptom", which may be noted:

```
"diagnose" imp("sickness" imp("syndrome"  
imp("symptom")))
```

or in the indented format:

```
1 diagnose  
  2 sickness  
    3 syndrome  
      4 symptom
```

Step 2, Instantiation

The printout of a particular instance of the schema is shown in Fig. 1. in form of "Deep Explosion" of "1 diagnose", i.e. the top-down recursive enumeration of its parts. ("Flat Explosion" displays just one level of parts.)

I-D3-5

Fig. 1. Deep Explosion of "diagnose".

```
1 diagnose axiom
  2 sickness_1_3 oof thrm
    3 syndrome_1_acf and thrm
      4 symptom_a and fact
      4 symptom_c and fact
      4 symptom_f and fact
    3 syndrome_3_bgh and thrm
      4 symptom_b and fact
      4 symptom_g and fact
      4 symptom_h and fact
  2 sickness_2_4 oof thrm
    3 syndrome_2_bcd and thrm
      4 symptom_b and fact
      4 symptom_c and fact
      4 symptom_d and fact
    3 syndrome_4_aeg and thrm
      4 symptom_a and fact
      4 symptom_g and fact
      4 symptom_e and fact
  2 sickness_3_4 oof thrm
    3 syndrome_3_bgh and thrm repetition
    3 syndrome_4_aeg and thrm repetition
```

Legend:

A. Numbers starting the lines are "levels" of the Structure. Node of level N implies directly nodes of level N+1:

"1 diagnose" implies "2 sickness_1_3",
"2 sickness_2_4", "2 sickness_3_4".

"2 sickness_1_3" implies "3 syndrome_1_acf",
"3 syndrome_3_bgh",

"3 syndrome_2_bcd" implies "4 symptom_b",
"4 symptom_c", "4 symptom_d".

I-D3-6

B."oof", "and" are Operators associated with relation between nodes of level N+1 and N:

1 syndrome_1_acf = and 2 symptom_a and
2 symptom_c and 2 symptom_f,

or in Polish Notation:

1 syndrome_1_acf = and (2 symptom_a,
2 symptom_c, 2 symptom_f)

In Polish Notation:

1 diagnose = oof(2 sickness_1_3, 2 sickness_2_4,
2 sickness_3_4) (is one of them).

C."axiom" denotes the top node which has parts, but no aggregates. In terms of the model it is an axiom, i.e. expression postulated arbitrarily as "certain" in the top-down deductive scan.

D."thrm" denotes theorems or middle nodes having parts and aggregates.

E."fact" denotes bottom nodes which have no parts and are set by extralogical events. In terms of the model they are principal premises of the inductive scan.

F."repetition" denotes a node whose explosion appears above in the indented display and is not repeated for conciseness' sake.

NOTE: the factual "symptoms" may be set by sensors of some technological device such as a space craft, or by a physician who observes symptoms of a patient, which are more or less typical, strong or in one word "plausible".
Fig. 2 shows a distribution of symptoms' plausibilities inputted to ERN for our tutorial.

I-D3-7

Fig. 2.

Input of symptoms' Plausibilities.

NOTE: All Plausibilities are expressed in percents.

symptom_a	and	98	fact
symptom_b	and	95	fact
symptom_c	and	96	fact
symptom_d	and	25	fact
symptom_e	and	18	fact
symptom_f	and	97	fact
symptom_g	and	98	fact
symptom_h	and	94	fact

Starting from those premises ERN executes the inductive, bottom up Inference scan, which evaluates the plausibilities of all nodes in function of those of facts-symptoms.

The results are shown in Fig. 3.

It's the same structure as that of Fig. 1 with additional, inductively evaluated Plausibilities.

I-D3-8

Fig. 3.

Deep Explosion of evaluated Instances.

```
1 diagnose 77 axiom
  2 sickness_1_3 oof 80 thrm
    3 syndrome_1_acf and 93 thrm
      4 symptom_a and 98 fact
      4 symptom_c and 96 fact
      4 symptom_f and 97 fact
    3 syndrome_3_bgh and 89 thrm
      4 symptom_b and 95 fact
      4 symptom_g and 98 fact
      4 symptom_h and 94 fact
  2 sickness_2_4 oof 1 thrm
    3 syndrome_2_bcd and 18 thrm
      4 symptom_b and 95 fact
      4 symptom_c and 96 fact
      4 symptom_d and 25 fact
    3 syndrome_4_aeg and 12 thrm
      4 symptom_a and 98 fact
      4 symptom_g and 98 fact
      4 symptom_e and 18 fact
  2 sickness_3_4 oof 6 thrm
    3 syndrome_3_bgh and 89 thrm repetition
    3 syndrome_4_aeg and 12 thrm repetition
```

Deep Explosion of the top node is in practical cases too long and too complex to be grasped at a glance. Even our very small example may be found not quite limpid. It is utile to navigate through the structure with help of one level or "flat" explosion, as shown in Fig. 4-6.

I-D3-9

Fig. 4. Flat Explosion of the top node "diagnose".

```
1 diagnose 77 axiom
  2 sickness_1_3 oof 80 thrm
  2 sickness_2_4 oof 1 thrm
  2 sickness_3_4 oof 6 thrm
```

"diagnose" implies "sickness_1_3", "sickness_2_4" and "sickness_3_4" whose Plausibilities are respectively 80,1,6. The Inductive Inference from the premises of Fig. 2 leads to the conclusion that "sickness_1_3" is by far the most plausible. The Plausibility of choosing "sickness_1_3" as one of ("oof") the three is evaluated in their Aggregate "diagnose" as 77.

Plausibility of the Axiom "diagnose" (77) represents acceptable inductive verification of the theory founded in it, embodied by the deduced ERN structure, and of the evaluation of three mutually exclusive ("oof" Operator) "sickness_...", suggesting the choice of "sickness_1_3".

Determination of the "oof" algorithm is discussed in Appendix.

Fig. 5.

Flat Explosions of the three "sickness".

```
1 sickness_1_3 80 thrm
  2 syndrome_1_acf and 93 thrm
  2 syndrome_3_bgh and 89 thrm
```

```
1 sickness_2_4 1 thrm
  2 syndrome_2_bcd and 18 thrm
  2 syndrome_4_aeg and 12 thrm
```

I-D3-10

1 sickness_3_4 6 thrm
2 syndrome_3_bgh and 89 thrm
2 syndrome_4_aeg and 12 thrm

We note that $\text{and}(93,89) = 80$; $\text{and}(18,12) = 1$;
 $\text{and}(89,12) = 6$;
Details of "and" algorithm is discussed in
Appendix.

Fig. 6. Flat Explosions of "syndromes".

1 syndrome_1_acf 93 thrm
2 symptom_a and 98 fact
2 symptom_c and 96 fact
2 symptom_f and 97 fact

1 syndrome_2_bcd 18 thrm
2 symptom_b and 95 fact
2 symptom_c and 96 fact
2 symptom_d and 25 fact

1 syndrome_3_bgh 89 thrm
2 symptom_b and 95 fact
2 symptom_g and 98 fact
2 symptom_h and 94 fact

1 syndrome_4_aeg 12 thrm
2 symptom_a and 98 fact
2 symptom_g and 98 fact
2 symptom_e and 18 fact

Explosion structures show for an Aggregate the
Parts it implies, either directly (Flat
Explosion) or recursively, till the bottom of
structure (Deep Explosion).

I-D3-11

One may be on the other hand interested for a Part by which Aggregates it's implied (to which inductive Conclusions it contributes), directly (Flat Implosion), or recursively till the top of structure (Deep Implosion). Fig. 7-8. show Flat and deep Implosion of "symptom_a".

Fig. 7. Flat Implosion of "symptom_a".

```
1 symptom_a 98 fact
  2 syndrome_1_acf and 93 thrm
  2 syndrome_4_aeg and 12 thrm
```

Fig. 8. Deep Implosion of "symptom_a".

```
1 symptom_a 98 fact
  2 syndrome_1_acf and 93 thrm
    3 sickness_1_3 and 80 thrm
      4 diagnose oof 77 axiom
  2 syndrome_4_aeg and 12 thrm
    3 sickness_2_4 and 1 thrm
      4 diagnose oof 77 axiom
  3 sickness_3_4 and 6 thrm
    4 diagnose oof 77 axiom
```

I-D3-12

D. Epistemological Conclusions

Epistemological impact of Relativistic Dialectic and its Logic, the ER Network, concerns mainly

- foundations,
- definitions and distinction of "Theory" and "Model",
- definitions and distinction of "Axiom" and "Dogma".

Foundations.

We have postulated that Logical Systems may be evaluated and justified exclusively by their capacity to simulate Mind's intrinsic, ER based Logic. ERN is the first Logical System founded in Mind's intrinsic Logic, rather than in noumenal linguistic expressions. It seems to simulate it efficiently, which has been verified by its several practical applications.

Theory and Model.

Contemporary Epistemology sees falsifiability as a necessary quality of scientific structures. ERN embodies it rigorously in its two complementary aspects:

1. Conceptual, deductive Theory,
2. Experimental, inductively falsifiable Model.

Axiom.

Full-fledged model structure supporting both, necessary deduction and fuzzy, factual induction will be called "axiomatic" and its top arbitrary presumptions - "Axioms".

Axioms and thence deduced Theory are falsifiable and refutable by inconclusive induction from factual experiments.

I-D3-13

Dogma.

A Theory lacking bottom factual Theorems and thus unable to support the falsifiable induction will be called "Dogmatic" and its top arbitrary presumptions - "Dogma". Unlike Axioms, Dogma are not falsifiable, cannot be refuted and repose in unshakable faith in transcendental "Truth".

I-D3-14

Appendix. Fuzzy Operators.

N Dimensions

As can be seen in the chapter "BOOLEAN SUPPORT OF ERN LOGIC" for N dimensions the Number of operators ($2^{(2^N)}$) increases very fast with N. For N=2 we had 16 operators which may be learnt by heart, like the multiplication table, so that with a bit of practice one can execute and program all operations of the 2D exact Calculus from memory. However, For N=4 we have $2^{(2^4)}=65536$ and for n=5 $2^{(2^5)}=2^{32}=4294967296$ operators. And 5 is small for practical applications. We may have 20 symptoms of a disease or 100 "symptoms" of some breakdown in a spacecraft. The respective diagnostic systems would extend over $2^{(2^{20})}$ and $2^{(2^{100})}$ operators. A bit too much to learn by heart, to describe in a textbook, or, for that matter, in the whole Congress Library. It's clear that for higher N's only a few operators can be chosen from endless lists in function of their utility for a particular problem. The user has to tailor his logic to his problem by choosing pertinent operators and designing their evaluation algorithms.

00F (One Of) Operator

Some Operators like "OR", or "AND" map from 2D to ND as one to one, but for instance the 2D Operator ORR ("exclusive or", "either-or") forks for ND to N distinct operators from "One Of" to "(N-1) Of" and "Not All" (see "BOOLEAN SUPPORT OF ERN LOGIC").

I-D3-15

For the Diagnose application we have retained from all bifurcating branches of ORR only the "00F" (One Of), as only one sickness may be chosen as base of subsequent therapy.

ERN offers to the expert the possibility of customizing the fuzzy Operators in function of application and expert's experience. For the Diagnose application we established the 00F algorithm as follows:

Meaningful cases encompass any number of Operands N greater than 1.

The values (in %) of concerned Operands are split into "MAX" (the maximum value, or first of equal greatest values in Operands' vector) and the rest.

SIG: sum of all N Operands.

The average of all but MAX:

$$\text{NOMAX} = (\text{SIG} - \text{MAX}) / (\text{N}-1)$$

and

$$\text{00F} = (\text{MAX} * (100 - \text{NOMAX})) / 100$$

For the ideal distribution (MAX=100, NOMAX=0) 00F = 100. With decreasing MAX and increasing NOMAX 00F decreases.

AND Operator

MIN: smallest value or first of equal smallest values in Operands' vector.

MED: Average of all concerned Operands.

$$\text{AND} = (\text{MIN} * \text{MED}) / 100$$

The apparently simple AND Operator has raised more discussions than any other one. The first approach is to treat it with the probability Multiplication Rule. For two rather certain Operands of 90% it seems reasonable to say that $\text{AND}(90,90) = 81$.

I-D3-16

Yet, the experts argued that if the progress of science discovers other 5 symptoms all confirmed in our instance at 90%, it should make the syndrome more certain, or at least equal and not disqualify it at 47% as the Multiplication Rule would do. Finally they accepted the above algorithm which maintains the syndrome at 81% for any number of 90% symptoms.