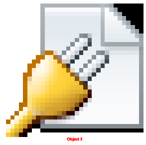

I-C4-1

# LIAR, RUSSELL AND GOEDEL

Liar's Paradox has been formulated in numerous forms, the simplest being "I always lie", supposed to loop infinitely: "I always lie", thus I lied, thus "I always tell truth", thus "I always lie" was true, thus "I always lie", etc.

Actually, it was no paradox at all but a sophism, coined at the time when Greeks discovered the delights of ingenious, unbridled reasoning, before conceiving ways to keep it under control.

In order to demystify the alleged paradox, we reformulate "I always lie" as equivalent, but more rigorous:
P="all propositions I utter are false"
and reconstruct the pretended loop:

A. I utter as true:
   P="all propositions I utter are false".
   thus
B. P is false
   thus
C. not-P="no propositions I utter is false"
   is true
   thus
D. P which I uttered is not-false, i.e. true
   and we are back at A.

However, since Liar's inception about 600 BC many ways to keep sophisms under control have been defined, starting with the Aristotelian Logic, which would clearly falsify C.

I-C4-2

Indeed, negation of the universal affirmative proposition
P="all propositions I utter are false"
does not give the universal negative proposition
not-P="no propositions I utter is false",
but the particular affirmative proposition
not-P="some propositions I utter are false"
and D. becomes
"P is either false or true", i.e. logically indeterminate.

Reconstructing correctly the chain of reasoning we get:

A. I utter as true:
   P="all propositions I utter are false".
   thus
B. P is false
   thus
C. not-P="some propositions I utter are false" is true
   thus
D. P is either false or true, i.e. logically indeterminate.

In other terms we don't know anything about P and Liar does not loop infinitely but peters out after the first correct conclusion.

I-C4-3

# Liar and the Established Logic

In "PREDICATE LOGIC" we have shown that the currently dominating Predicate Logic (PL) is a noumenal, ill-founded, instance of Naive View. Russell's paradox shuttered PL with its feeble foundations, but logicians did not dare to abandon the Naive View and tried vainly to save and rebuild the collapsed structure upon its rotten base rather than construct a new edifice upon new rational foundations.

As result, the established Logic became an extraordinary proliferation of competitive remedies of PL, all noumenal, ill-founded and meaningless. We have counted about 100 of them. There are also about as many non-PL systems, mainly based upon corrupted Boolean Algebra ("BOOLEAN SUPPORT OF ERN LOGIC") as meaningless and useless as PL remedies, but we shall consider here only the latter, as on the one hand they constitute the mainstream founding the official Set Theory and, on the other hand, they reserve an eminent place for Liar's Paradox. Indeed, after 2000 years of quiet rest at the cemetery of deduction errors it resurrected, donned the old moth eaten dress of Paradox and made a triumphal come-back as a principal hinge of most, if not all PL remedies. We don't intend to discuss all 100+ of them, but shall have a short look on two leading and typical "applications" of Liar, those of Russell and Goedel.

# Russell

In his Types Theory Russell introduces the notion of first-order, second-order and higher order logics in this way:

...We may define an individual as something destitute of complexity; it is then obviously not a proposition, since propositions are essentially complex. Hence in applying the process of generalization to individuals we run no risk of incurring reflexive fallacies. Elementary propositions together with such as contain only individuals as apparent variables we will call first-order propositions. We can thus form new propositions in which first-order propositions occur as apparent variables. These we will call second-order propositions; these form the third logical type.

The super-naive Ontology underlying these assertions is discussed in "PREDICATE LOGIC". Here we are mainly interested in surprising resurgence of Liar's "Paradox" in the monumental Principia Mathematica often considered as the principal contribution to foundations of Logic and Mathematics:

-Thus, for example, if Epimenides asserts "all first-order propositions affirmed by me are false," he asserts a second-order proposition; he may assert this truly, without asserting truly any first-order proposition, and thus no contradiction arises.-

If Russell studied Aristoteles, he would have noticed that the "contradiction" disappeared 2000 years ago, reduced to confusion of a particular affirmative proposition with a universal negative one.

This would dispense him from writing thick fallacious treaties and us from being muddled.

But things went still much deeper. Liar had many shades, one of them, the Eubulides "paradox": "This statement is false, thus it's true, thus it's false", etc. Russell treated it seriously and even went to the trouble of creating its two statement version: "The following statement is true. The preceding statement is false".

Talking about it one feels embarrassed like listening to a fellow who had one over the eight and laughs heartily at his own silly jokes.

Symbolizing "This statement is false" with "R", R is neither false, nor true for the simple reason that it is NO statement at all.

By the standards of Russell's own Predicate Logic a statement is a predication, an assignment of a property to a subject. "Truth/Falsity" qualifies the predication itself and not the subject of predication.

A statement "all cars are red" is a valid predication or statement which may be true or false and by virtue of observations turns out to be false. Now, R does not assign any property to any subject, thus is not a predication, not a statement at all, a meaningless chain of characters that may not be true or false.

As meaningless as the built upon it Types Theory.

I-C4-6

# Goedel

-For any consistent formal theory that proves basic arithmetical truths, it is possible to construct an arithmetical statement that is true but not provable in the theory. That is, any theory capable of expressing elementary arithmetic cannot be both consistent and complete.-

Whatever the meaning, if any, of "basic arithmetical truths" may possibly be, we shall recall how Goedel describes his "theory" and the famous "G" (Goedel Sentence) true but not provable in the theory.

"Theory" refers to an (infinite) set of statements, some of which are taken as true without proof (these are called axioms), and others (the theorems) are taken as true because they are provable from the axioms. "Provable in the theory" means "derivable from the axioms and primitive notions(?) of the theory, using standard first order logic." A theory is "consistent" if it never proves a contradiction. "It is possible to construct" means that there exists some mechanical(?) procedure which can construct the statement, given the axioms, primitives, and first order logic. The resulting true but unprovable statement is often referred to as "the Goedel sentence" for that theory. In fact, there are infinitely many statements in the theory that share with the Goedel sentence the property of being true but not provable from the theory. "Elementary arithmetic" consists merely(?) of addition and multiplication over the natural numbers.

I-C4-7

We have seen in "PREDICATE_LOGIC" and "SET_THEORY" the inanity of founding mathematics and mathematical proofs in "standard first order logic" and more generally, in discreteness. For an arithmetic theory properly founded in continuous geometry, the question of completeness could not arise. If consistently reducible to geometric continuum, it would discretize only a partial area thereof and thus be incomplete; otherwise, it would be inconsistent.

But let's consider for oddity's sake the famous Goedel sentence G: "This sentence is not provable" and the Olympian theorem:
"G is true but not provable in the theory".
G cannot be false, because then it would be provable and all provable sentences are true; so it's necessarily true and unprovable).

Now, as Russell's "R", Goedel's "G" is a shade of the exhumated Liar's "Paradox" and no "sentence" at all.

Why did Goedel play amateur gravedigger and disinter the poor Liar, when he had any number of true and unprovable axioms at hand, will stay for ever a closed book.

Unless, as some people say, he considered axioms as provable from the theory. But it must be a malignant calumny. Even Goedel could not be that inane.

At least...

In "Goedel's Proof" by Peter Suber, Philosophy Department, Earlham College,we read:

***
Suppose we added G to the axiom set of S. Then G would become provable, since all axioms are provable by definition.
***

The mind boggles. Since 2300 years all scientific theories are axiomatic and their founding axioms are unanimously considered as "by definition" true and unprovable. So Suber's "definition" barring the whole scientific history seems a bit exotic and one would associate it rather with some exclusive loony bin than with science, mathematics, logic, or anything rational.

And Mr Suber continues:

***
Goedel proved that if we do this (add G to the axiom set of S), creating S', then we could always construct another undecidable wff, G', which asserted that it could not be proved in S'. Of course we could then add G' to S', creating S'', but then we could construct G'', and so on ad infinitum.
***

Very exclusive must indeed be the loony bin where Peter Suber may venerate his co-certified master Goedel confusing mathematics with russian dolls.

This Goedel's betise may be added to all those shown in the chapter "PREDICATE LOGIC".